

Solution to Quiz 1, STAT 5685 — Mathematical Statistics II, Spring 2009

Name:

Points:

1. (10 points) Suppose that X_1, X_2, X_3 are iid Bernoulli variables with unknown success rate θ , $0 < \theta < 1$.
- (a) (5 points) Find a scalar sufficient statistic and justify.
- (b) (5 points) Show that $U = X_1X_2X_3$ is not sufficient.

Solution:

- (a) Bernoulli is an exponential family. Therefore, $T = X_1 + X_2 + X_3$ is sufficient.
- (b) It suffices to show that there exist some x_1, x_2, x_3 and u such that $\Pr(X_1 = x_1, X_2 = x_2, X_3 = x_3 | U = u)$ depends on θ . Consider, for example, the case where $X_1 = 1, X_2 = 0, X_3 = 0$ and $U = 0$.

□

2. (10 points) Continue with the last problem.
- (a) (5 points) Find a minimal sufficient statistic and justify.
- (b) (5 points) Show that $V = (X_1 + X_2, X_3)$ is sufficient but not minimal sufficient.

Solution:

- (a) Bernoulli is an exponential family. Therefore, $T = X_1 + X_2 + X_3$ is minimal sufficient.
- (b) It suffices to show that T is a function of V but not a one-to-one function of V . First, $T = V_1 + V_2$ is a function of V . Second, T is a many-to-one function of V .

□

THE END