

Quiz 2 Solution, STAT 5685 — Mathematical Statistics II, Spring 2009

Name:

Points:

1. (10 points) Suppose that X_1 and X_2 are iid $N(\theta, 1)$.

(a) (5 points) Let \vec{X} be the sample vector. Find $I_{\vec{X}}(\theta)$.

(b) (5 points) Let $T_p = X_1 + pX_2$ for some known $p > 0$. Find $I_{T_p}(\theta)$.

Solution:

(a) $I_{\vec{X}}(\theta) = 2I_{X_1}(\theta) = 2$.

(b) Note that T_p follows $N(\theta(1+p), 1+p^2)$. Use definition of information to find $I_{T_p} = (1+p)^2/(1+p^2)$.

□

2. (10 points) Suppose that X_1, \dots, X_n form a random sample from $N(\theta, \theta)$, $\theta > 0$.

(a) (5 points) Find a complete and sufficient statistic.

(b) (5 points) Let S_n^2 be the sample variance and $X_{n:1}, \dots, X_{n:n}$ be the order statistics. Show that $\sum_{i=1}^n X_i^2$ and $\frac{X_{n:n} - X_{n:1}}{S_n}$ are independent.

Solution:

(a) From results for exponential family, $\sum_{i=1}^n X_i^2$ is complete and sufficient.

(b) Establish ancillaries and use Basu's theorem.

□

THE END