Quiz 3, STAT 5685 — Mathematical Statistics II, Spring 2009

Name: **Points:**

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1. (10 points) Suppose that $X_1, \ldots, X_n$ are iid from Poisson distribution with mean θ. Let $\bar{X}_n$ be the sample mean.

   (a) (5 points) Find the UMVUE of θ with justification.

   (b) (5 points) Find $E[X_1 + X_2 | \bar{X}_n]$.

   Solution:

   (a) From results for exponential family, $\bar{X}_n$ is complete and sufficient. Since $E[\bar{X}_n] = \theta$, by Lemman-Sheffe's Theorem, $\bar{X}_n$ is the UMVUE of θ.

   (b) The desired conditional expectation is the UMVUE of $E[X_1 + X_2] = 2\theta$, $2\bar{X}_n$.

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2. (10 points) Suppose that $X_1, \ldots, X_n$ are iid from $U(0, \theta)$, where $\theta > 0$ is unknown. Let $X_{n:n}$ be the sample maximum. It is known that $X_{n:n}/\theta$ is a Beta$(n, 1)$ random variable, with mean $n/(n+1)$ and second moment $n/(n+2)$. Consider estimating θ.

   (a) (4 points) Find the MSE of estimator $cX_{n:n}$ for constant $c > 0$.

   (b) (3 points) Find the optimal c which minimizes the MSE of $cX_{n:n}$.

   (c) (3 points) Is the optimal $cX_{n:n}$ consistent for θ? Justify.

   Solution:

   (a) $E[(cX_{n:n} - \theta)^2] = c^2 E[X_{n:n}^2] - 2c\theta E[X_{n:n}] + \theta^2 = \theta^2 [c^2 \frac{n}{n+2} - 2c\frac{n}{n+1} + 1]$.

   (b) Minimizing the quadratic of c yields optimal $c^* = \frac{n+2}{n+1}$.

   (c) $\frac{n+2}{n+1} X_{n:n} \xrightarrow{p} \theta$ by Slutsky's theorem, $X_{n:n} \xrightarrow{p} \theta$, and $\frac{n+2}{n+1} \to 1$.

   □

THE END