

Quiz 4, STAT 5685 — Mathematical Statistics II, Spring 2009

Name:

Points:

1. (10 points) Suppose that X_1, \dots, X_n are iid from Weibull distribution with pdf

$$f(x; a) = a^{-1}bx^{b-1} \exp(-x^b/a), \quad x > 0,$$

where $a > 0$ is unknown and $b > 0$ is known.

- (a) (5 points) Show that the distribution has MLR property in a minimal sufficient statistic.
(b) (5 points) Find the UMP test of level α for $H_0 : a \leq 1$ vs $H_1 : a > 1$.

Solution:

- (a) With b known, the distribution belongs to a one-parameter exponential family and therefore has MLR nondecreasing in minimal sufficient statistic $T = \sum_{i=1}^n X_i^b$.
(b) By Karlin-Rubin's Theorem, the UMP test rejects H_0 if $T > k$, with k determined by level α . Note that, under H_0 , the distribution of T is $\Gamma(n, 1)$. Therefore, k is the $(1 - \alpha)$ quantile of $\Gamma(n, 1)$.

□

2. (10 points) Suppose that X_1, \dots, X_n are iid from exponential distribution with mean $\theta > 0$.

- (a) (5 points) Find the MLE of θ .
(b) (5 points) Construct a LR test of level α for $H_0 : \theta = 1$ vs $H_1 : \theta \neq 1$.

Solution:

- (a) Solve the score equation and check the second derivative to obtain $\hat{\theta} = \sum_{i=1}^n X_i/n$.
(b) LR is

$$\Lambda = \frac{\exp(-\sum X_i)}{\left(\frac{\sum_{i=1}^n X_i}{n}\right)^{-n} \exp(-n)}.$$

LR test rejects H_0 if $\Lambda < k$, or equivalently, $\sum_{i=1}^n X_i > b$ or $\sum_{i=1}^n X_i < a$. Note that $\sum_{i=1}^n X_i$ is $\Gamma(n, 1)$ under H_0 . Therefore, a and b can be chosen to be the $\alpha/2$ and $1 - \alpha/2$ quantiles of $\Gamma(n, 1)$.

□

THE END